Optics B.Sc. Part II Hons Physics

Format's Principle and lens and Mirror formula

Format's Principle → A ray of light in passing from one point to another by any number of reflections or refraction choose a path of least time.

On this principle the law of rectilinear propagation, the laws of refraction and reflection can be derived. However in some cases it has been found that the time taken by the light is not minimum but maximum or else it is neither maximum nor minimum, but is stationary. This is found in the case of formation of image in the case of lens in which all rays starting from an object point in reaching the image point is the path of maximum or minimum time. This is known as Fermat's principle of stationary time.

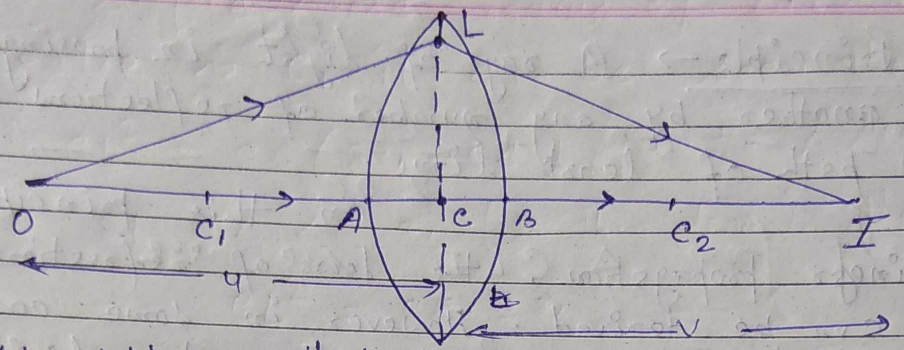
For a homogeneous and isotropic medium the refractive index 'μ' is constt. and hence the optical path traversed is minimum.

For a non-homogeneous and an isotropic medium the optical path traversed $\mu_1 S_1 + \mu_2 S_2 + \mu_3 S_3 - - - -$ is minimum or maximum or stationary.

Thus there is another way of stating format's principle.

For refraction through prisms and lenses the path of the ray between two points is that for which the time is stationary in value.

Refraction at a lens :-



Let us suppose that L is a convex lens whose refractive index is μ. Let O be the point object placed on the axis of the lens at a ~~displace~~ distance u from the optic centre C i.e. OC = u

A ray OL strikes the lens at L (approx). and after refraction converges at the point I. Another ray OA directed along the axis of the lens goes undeviated after refraction and cuts the first refracted ray LI at I such that CI = v. Then I is the image of O.

Since the rays starting from O intersect at I, the optical paths along the two directions according to the law of extreme paths, must be the same and hence $OL + LI = OA + \mu.AB + BI =$ max; min; or const. (1)

from Δ OLC, we have

$$OL^2 = OC^2 + CL^2$$

$$= OC^2\left(1 + \frac{CL^2}{OC^2}\right)$$

$$\therefore OL = OC\left(1 + \frac{CL^2}{OC^2}\right)^{1/2}$$

from Binomial expansion we have

$$OL = OC\left(1 + \frac{1}{2}\cdot\frac{CL^2}{OC^2}\right)$$

$$\text{or, } OL = OC + \frac{CL^2}{2\,OC} \quad \text{—— (2)}$$

Similarly from $\Delta\, CLI$, we have

$$LI^2 = CI^2 + CL^2$$

$$= CI^2\left(1 + \frac{CL^2}{CI^2}\right)$$

$$\therefore LI = CI\left(1 + \frac{CL^2}{CI^2}\right)^{1/2}$$

$$\text{or, } LI = CI + \frac{CL^2}{2\,CI} \quad \text{—— (3)}$$

Putting the values of OL and LI in eqn (1), we get:

$$OC + CI + \frac{CL^2}{2}\left(\frac{1}{OC} + \frac{1}{CI}\right) = OA + \mu\cdot AB + BI$$

$$\text{or, } \cancel{OC} + \cancel{CI} + \frac{CL^2}{2}\left(\frac{1}{OC} + \frac{1}{CI}\right) = \cancel{OC} - AC + \cancel{CI} - BC + \mu(AC + BC)$$

$$\therefore \frac{CL^2}{2}\left(\frac{1}{OC} + \frac{1}{CI}\right) = \mu AC - AC + \mu BC - BC$$

$$= (\mu - 1)AC + (\mu - 1)BC$$

$$= (\mu - 1)(AC + BC)$$

$$\therefore \frac{1}{OC} + \frac{1}{CI} = \frac{2(\mu - 1)(AC + BC)}{CL^2} \quad \text{—— (4)}$$

For the surface whose radius of curvature is r_1 and centre is C, it can be seen that

$$CL^2 = CB(2r_1 - CB) \quad \text{(from the property of circle)}$$

If $CB << r_1$ (for thin lens), then

$$CL^2 = CB \cdot 2r_1 \quad \text{--- (5)}$$

Similarly, $CL^2 = CA \cdot 2r_2$ --- (6)

from eqn (4), we get

$$\frac{1}{OC} + \frac{1}{CI} = 2(\mu-1)\left(\frac{AC}{CL^2} + \frac{BC}{CL^2}\right)$$

or, $$\frac{1}{u} + \frac{1}{v} = 2(\mu-1)\left(\frac{AC}{AC\cdot 2r_2} + \frac{BC}{BC\cdot 2r_1}\right)$$

or, $$\frac{1}{u} + \frac{1}{v} = (\mu-1)\left(\frac{1}{r_2} + \frac{1}{r_1}\right) \quad \text{--- (7)}$$

Applying sign convention, Hence eqn (7) becomes:

$$\frac{1}{v} - \frac{1}{u} = (\mu-1)\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \quad \text{--- (8)}$$

If 'f' be the focal length of the lens, then

$$\frac{1}{f} = (\mu-1)\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \quad \text{--- (9)}$$

This is lens formula.

from eqn (8), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{--- (10)}$$

This is the required lens formula for the refraction at a lens.

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